

# *Schrödinger Theory on Time Scales*

Gaussian Bells, Corresponding Banach  
Spaces and the  $\mathbb{T}$ -Harmonic Oscillator

von

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